Components of a Synchrotron

• The electron storage ring (accelerator physics)
• The experimental beamlines (users)
  - Front-end
  - Optics
  - End-stations

Ideally, the design of an experiment starts at designing the storage ring
Emittance: $\varepsilon$

The brightness of the light depends on how *tightly the electron beam is squeezed.*

Spatial deviation of the electron from the ideal orbit is $\sigma_x$, in the plane of the orbit with angular spread $\sigma_x'$. $\sigma_y$, vertical to the plane of the orbit, with $\sigma_y'$. Emittance $\varepsilon$(nm-rad) is expressed as

$$\varepsilon_x = \sigma_x \sigma_x'$$
$$\varepsilon_y = \sigma_y \sigma_y'$$

Emittance is conserved (Liouville theorem)

Typical emittance of 3rd generation ring: 10 - 1 nm-rad
Energy loss to synchrotron radiation

\[ P = \frac{2}{3} \frac{e^2 \gamma^2}{m_0^2 c^3} \left| \frac{dp}{dt} \right|^2 \]

\[ v \sim c, \ pc = E \]
\[ \gamma = \frac{E}{m_0 c^2} \]

\[ P = \frac{2}{3} \frac{e^2 c}{(m_0 c^2)^4} \frac{E^4}{\rho^2} \]

\[ \Delta E = \int_{\text{orbit}} P \, dt = P \frac{2 \pi \rho}{c} \]

\[ \Delta E [\text{keV}] = 88.5 \frac{E^4 [\text{GeV}^4]}{\rho [\text{m}]} \]

Energy loss per turn per electron \(\rightarrow\) synchrotron radiation!

For a 3.5 GeV ring with a radius of 12.2 m, the energy loss per turn is \(~10^6\) eV \(\Rightarrow\) Most storage rings built to-day are \(~\) GeV rings
How is synchrotron radiation generated?

- There are three ways
  - *To bend* the electron with a bending magnet (dipole radiation)
  - *To wiggle* the electron with a periodic magnetic field called insertion device
    - high field with a big bent: wigglers
    - medium field with many small bends: undulator
Synchrotron radiation from BM
BM (dipole) spectrum and critical energy

Critical energy $E$ (eV)

half of the radiated power is into photons with energy above $E_c$, and half of the power is below

$$E(eV) = \frac{12398.5}{\lambda (\text{Å})}$$

$$\lambda_c = \frac{12386.5}{800(eV)} (\text{Å}) = 15.5 \text{ Å}$$
Wigglers and undulators

Alternating magnetic structures

*short period, large bend: wiggler*
*long period, small bend: undulator*

\[ K = 0.934 \lambda_u [\text{cm}] B_0 [\text{T}] \]

strength parameter

peak magnetic field

angle of deflection

\[ \delta = K / \gamma \]
Undulator equation

\[ \lambda_1(\Theta) = \frac{\lambda_u}{2\gamma^2} \left[ 1 + \frac{K^2}{2} + \gamma^2 \Theta^2 \right] \]

\[ \Theta: \text{angle of observation} \]

\( K \gg 1 \): large field, sizable bend, negligible interference \( \Rightarrow \) wiggler

\( K < 1 \): short period, modest bend, interference \( \Rightarrow \) undulator

Fundamental energy/wavelength on axis

\[ \epsilon_1[keV] = \frac{0.950 E^2[GeV]}{(1 + K^2 / 2) \lambda_u[cm]} \]

\[ \lambda_1\left[ \frac{o}{A} \right] = \frac{13.06 \lambda_u[cm](1 + K^2 / 2)}{E^2[GeV]} \]
Modern Insertion Devices (APPLEII)

- Energy tunable by adjusting the gap
- In vacuum small gap undulator
- Polarization tunable by tuning the magnetic structures: EPU

The magnetic structure of the APPLE II consists of two pairs of arrays of permanent magnets.
Chicane Insertion Device at CLS
Characteristics of synchrotron radiation

- **Spatially distribution**: highly collimated half angle \( \psi = 1/\gamma, \gamma = 1975 \text{ E (GeV)} \).
  - bending magnet: \( 1/\gamma \)
  - undulator: \( 1/(\gamma \sqrt{N}) \)
  - wiggler: \( >> 1/\gamma \)

- **Spectral distribution**: continuous at BM and wiggler sources, spike like peaks in undulator source due to interference effects
Spatial distribution

Bending Magnet — A "Sweeping Searchlight"

Wiggler — Incoherent Superposition

Undulator — Coherent Interference

Spectral distribution

Advanced Light Source
1.3 GeV, 400 ma
Undulator D, 142 Poles
λ_u = 3.5 cm, B_o = 0.57 Tesla

Average Spectral Brilliance
Photons (sec)(mm^2)(mm^2)(0.1% Bandwidth)

Photon Energy (keV)
Why is synchrotron radiation special For materials analysis?

- Tunability (IR to hard x-rays)
- Brightness (highly collimated)
- Polarization (linear, circular, tunable)
- Time structure (short pulse)
- Partial coherence (laser-like beam from ID)
0.1% band path

100 eV

99.9 eV \rightarrow 100.1 eV
Brightness and flux

- **Brightness**: photons per 0.1% bandwidth sec\(^{-1}\) mm\(^{-2}\) mrad\(^{-2}\) mA\(^{-1}\)
- Number of photons per bandwidth, per unit time per **unit source area**, per unit **solid angle** (inherent to ring design: *emittance*)
- **Flux**: photons per 0.1% bandwidth sec\(^{-1}\)mA\(^{-1}\)
  Number of photons per bandwidth per unit time (scaled to ring current and orbit of arc, *can be improved by beamline optics*)
Brightness and Flux revisited

**Brightness:**
No. of photons per sec per mm$^2$ (source size) per mrad$^2$ (source divergence) per mA, within a 01.% band width

**Flux:**
No. of photons per sec per mrad angle of arc within a 01.% band width
Spectral Distribution of a BM source

BM radiation has a wide energy distribution characterized by of $\lambda_c$

$$\lambda_c (\text{Å}) = \frac{18.6}{E^2 B} = \frac{5.6 \rho}{E^3} = \frac{4\pi \rho}{3\gamma^3}$$

*note*: $\gamma = 1957E(\text{GeV})$

Example: SRC (WI)

E$= 0.8$ GeV,
$\rho = 2.0833$ m,
$\lambda_c = 22.7$Å

$E_c = 12398.5/22.7 = 546$ eV

Exercise: Comment on the similarity between Bremstrahlung and SR
Why is there a wide spread in photon energy?

Qualitative understanding: Uncertainty Principle

Pulse width \( \Delta t = t_e - t_\gamma \)

 Photon travels faster than electron

\[
\Delta t = t_e - t_\gamma = \frac{2\rho}{\beta \gamma c} - \frac{2\rho \sin(1/\gamma)}{c} \approx \frac{4\rho}{3c\gamma^3}
\]

Exercise: Show this; hint: \( \gamma \gg 1 \); \( \sin x = x - x^2/3! + \ldots \)

\[\Delta E \cdot \Delta t \geq \hbar/2\]

\[\Delta E_{SR} \geq \hbar/2\Delta t \geq 3\hbar c \gamma^2/2\rho\]

Typically \( \sim \) keV
The opening angle: angular distribution

Opening angle, $\psi$, is defined as the half angle of the SR above and below the orbit plane.

$$\psi \sim \left(\frac{1}{\gamma}\right) 0.57 \left(\frac{\lambda}{\lambda_C}\right)^{0.43} \text{ (radian)}$$

$$\psi \sim \left(\frac{1}{\gamma}\right)$$

We recall that $\gamma = 1957 \ E(\text{GeV})$, hence high energy electrons will produce a more **collimated** photon beam. Also the shorter the wavelength the more collimated the light. e.g.: X-ray is more collimated than IR
Polarization

Degree of polarization

\[ p = \frac{I_\parallel - I_\perp}{I_\parallel + I_\perp} \]

\[ \psi \sim \left(\frac{1}{\gamma}\right) 0.57 \left(\frac{\lambda}{\lambda_C}\right)^{0.43} \] (radian)

\[ \psi \gamma \sim 0.57 \left(\frac{\lambda}{\lambda_C}\right)^{0.43} \]

Polarization increases at shorter wavelength
The radio-frequency cavity

Where is a cavity located in a storage ring?
At a straight section

What does it do?
To replenish the energy loss by electrons as synchrotron radiation

How does it work?
To provide a time varying electric field (voltage) that gives the electron a boost when it passes the accelerating gap (resonant cavity)
The bunching effect of the r.f. cavity

Cavity voltage varies sinusoidally with a phase angle $\phi$. Since $(\phi/2\pi) \cdot$ frequency $=$ time, the $\phi$ axis is equivalent to time

$\phi_s$ (stable phase angle): the electron receives the correct amount of energy it lost to synchrotron radiation

If an electron passes through the gap late ($\phi_1$), it will not receive sufficient energy and moves into an orbit of smaller radius and will take less time to arrive back at the gap. Thus electrons are said to be bunched by the r.f. cavity.

The potential well within which they are stable is called the **bucket**.
Time Structure

r.f. cavity bunches the electrons. Bunch length determines the pulse width

Number of bunches determines the repetition rate

ALS, 1.9 GeV
328 buckets, bunch: 35 ps
circumference = 197 m
197/328 = 0.6 m
Separating the bunches
0.6m/3x10^8 ms^{-1} = 2 ns
# Relevant storage ring parameters (CLS)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Main hardware</th>
<th>Relevance to users</th>
</tr>
</thead>
</table>
| **Energy** (e.g. 2.9GeV) | B field and radius  

\[
\lambda_c (\text{Å}) = \frac{18.6}{E^2 B} = \frac{5.6 \rho}{E^3}
\]

practical photon energy (\(\lambda_c\)) |
| **Current** (500 mA) | r.f. cavity: time varying voltage | photon flux |
| **Emittance** (16 nm-mrad) | magnetic lattice | Brightness (microbeam) |
| **Circumference** (8 straight sections) | r.f. cavity (500 MHz), ID | time structure (dynamics), microbeam |
The magnetic system (lattices)
The system of magnetic optics that guides and focuses an electron beam is called the lattice. The choice of the lattice is the most critical decision for it determines

Emittance (electrons) – brightness (photon beam)

Beam lifetime
and stability

No. of insertion devices

Cost
Magnetic Lattice

Lattice Characteristics
The cell: building block arrangement of bending magnet and focusing magnets
a. Double bend Achromat
b. Triple bend Achromat

New: Multiple bend achromat
The Aladdin Ring (Stoughton Wisconsin): Home of the Canadian Synchrotron Radiation Facility triple bend

UV Ring NSLS, Brookhaven national Lab

1 Focussing Quadrupole
2 Defocussing Quadrupole
3 Sextupole Correction
5 Accelerating Cavity
9 Straight Sections (available for Undulators and Wigglers)
7 Injection Kicker
8 100 MeV Microtron

double bend
The journey of the electron in the storage ring

Electron gun → Linac → Booster

How a Synchrotron Works

4. Storage Ring
The booster ring feeds electrons into the storage ring, a many-sided donut-shaped tube. The tube is maintained under vacuum, as free as possible of air or other stray atoms that could deflect the electron beam. Computer-controlled magnets keep the beam absolutely true.

Synchrotron light is produced when the bending magnets deflect the electron beam; each set of bending magnets is connected to an experimental station or beamline. Machines filter, intensify, or otherwise manipulate the light at each beamline to get the right characteristics for experiments.

5. Focusing the Beam
Keeping the electron beam absolutely true is vital when the material you’re studying is measured in billions of a metre. This precise control is accomplished with computer-controlled quadrupole (four pole) and sextupole (six pole) magnets. Small adjustments with these magnets set to focus the electron beam.

3. An Energy Boost
The linac feeds into the booster ring which uses magnetic fields to force the electrons to travel in a circle. Radio waves are used to add even more speed. The booster ring ramps up the energy in the electron stream to between 1.5 and 2.9 gigaelectron volts (GeV). This is enough energy to produce synchrotron light in the infrared to hard X-ray range.

2. Catch the Wave
The electron stream is fed into a linear accelerator, or linac. High energy microwaves and radio waves chop the stream into bunches, or pulses. The electrons also pick up speed by “catching” the microwaves and radio waves. When they exit the linac, the electrons are traveling at 99.9998% of the speed of light and carry about 300 million electron

1. Ready, Aim...
Synchrotron light starts with an electron gun. A heated element, or cathode, produces free electrons which are pulled through a hole in the end of the gun by a powerful electric field. This produces an electron stream about the width of a human hair.

Source: University of Saskatchewan / Paradigm Media Group Inc.
Beamlines and experimental stations

Beamline optics solution:

1. Energy region (monochromator)
2. Photon intensity vs. resolution (slits and mirrors)
3. Spatial resolution (emittance, focusing)
4. Polarization (undulator)
5. Coherence (undulator)
Optical elements

a. **Aperture/slits** (resolution and flux trade-off)
b. **Mirrors**
   (collimation, focus, higher order rejection)
a. **Monochromators** (monochromatic light)
   IR- gratings, FTIR (interferometer)
   UV, VUV (3-100 eV) – gratings
   VUV- soft x-ray gratings (100 eV -5000 eV) and crystals with large 2d values, e.g. InSb(111)
   Hard x-rays – crystals (Si(111), Si (220) etc.)
X-ray Mirrors

(a) Refraction and reflection of light and X-rays

- Plane mirror: collimation;
- Curved mirror (spherical, elliptical, etc.): focusing

The index of refraction for X-rays is slightly less than 1

$\theta_c$: critical angle where total reflection occurs

(b) Focusing X-ray mirror

Plane mirror: collimation;
Curved mirror (spherical, elliptical, etc.): focusing
Mirror and X-ray reflectivity

Let $\theta_c$ be the angle at which total reflection occurs, then

$$\sin \theta_c = \lambda (N r_0 \pi)^{1/2}$$

N: # of electrons per cm$^3$

at glancing angle  $\theta_c \sim \sqrt{\delta} \sim (2.74 \times 10^{-6} Z \rho A)^{1/2} \lambda$

atomic #, density, atomic mass

C K-edge @ 290eV

high energy photon cutoff

decreasing angle of incidence

1 rad = 57.3 $^\circ$, 1$^\circ$ = 17.45 mrad

Photon energy (eV)
Reflectivity as a function of incident energy and angle

The energy cutoff at 0.5 ° angle of incidence is ~ 4 keV; this property can be used to filter out high energy photons (higher order).
What is a grating and a grating monochromator?

Monochromatic light can be selected with a moving aperture or by rotating the grating

Grating: arrays of lines with well defined separation and profile
Consider a plane grating, the grating equation is

\[ n\lambda = d(sin\alpha + sin\beta) \]

n: order of the diffraction; d: distance between the grating lines;
d = W/N where W is the ruled width and N the no. of lines.

A 800 line grating means 800 lines per mm

\( \alpha: \) angle of incidence  \( \beta: \) angle of diffraction

zero order position, \( \alpha = -\beta \)
zero order normal

rotation

\[ \Phi: \text{angle of rotation of grating away from the zero order position} \]

\[ \theta: \text{angle between zero order normal and incident beam} \]

\[ \alpha = \theta - \phi, \quad \beta = -\theta + \phi; \quad n\lambda = 2d \cos\theta \sin\phi \]

The resolution: \[ \frac{E}{\Delta E} = \frac{\lambda}{\Delta \lambda} \propto N_l n \]

It can be seen high resolution can be obtained with high line density (N_l) grating or the use higher order (n) radiation. E.g.: A 1800 line grating has better resolution than a 1200 line grating.
Double Crystal Monochromator (DCM)

Bragg’s law $n\lambda = 2d \sin \theta$

- $n$: order; $\lambda$: wavelength;
- $d$: lattice spacing; $\theta$: Bragg angle

The resolution:

$$\frac{\Delta \lambda}{\lambda} = \cot \theta \Delta \theta$$

$\Delta \theta$ depends on the inherent width of the crystal (Darwin curve/rocking curve) and the vertical angular spread of the SR

$$\Delta \theta = \sqrt{\Delta \theta^2_{SR} + \Delta \theta^2_C}$$

$\Delta \theta_{SR} = \psi \sim \left(\frac{1}{\gamma}\right)^{0.57} \left(\frac{\lambda}{\lambda_C}\right)^{0.43} \text{ (rad)}$

For most crystals $\Delta \theta_C > \Delta \theta_{SR}$. Crystals used for DCM

<table>
<thead>
<tr>
<th>Crystal</th>
<th>Bragg Reflection</th>
<th>$2d(\text{Å})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>(111)</td>
<td>6.271</td>
</tr>
<tr>
<td>InSb</td>
<td>(111)</td>
<td>7.481</td>
</tr>
</tbody>
</table>
Crystal planes and Miller indices

Miller indexes \((h,k,l)\) are used to define parallel planes in a crystal from which the inter-planar spacing \(d\) can be obtained.

For a cubic crystal, the Miller indices \((111)\) refers to the set of planes that is parallel to the plane that intercepts the three axes at \(x = a, y = a\) and \(z = a\).

\[
d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}
\]

Exercise: work out the lattice spacing for Si\((111)\) shown above.
What is a Darwin (rocking) curve?

(a) Darwin curve of Si(111), reflectivity is 100% for \( x \) between -1 and 1,

(b) Darwin curve of Si(111) as a function of rotation angle @ 3 energies

(c) Darwin curve of Si(333) with different polarization

\[ W = \zeta \tan \theta \]

| \( \Delta \theta_c = w = \zeta \tan \theta \) (mrad) |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \( hv = 8050 \text{ eV} \) | (111) \( \tan \theta = 0.2534 \) | (220) \( \tan \theta = 0.4379 \) | (400) \( \zeta = 26.3 \times 10^{-3} \text{ mrad} \) |
| Si \( a = 5.4309 \text{ \AA} \) | \( \zeta = 139.8 \times 10^{-3} \) | \( \zeta = 61.1.8 \times 10^{-3} \) | \( \zeta = 26.3 \times 10^{-3} \) |
| Ge \( a = 5.6578 \text{ \AA} \) | \( \zeta = 347.2 \times 10^{-3} \) | \( \zeta = 160.8 \times 10^{-3} \) | \( \zeta = 68.8 \times 10^{-3} \) |